

# Caching

# memory hierarchy assumptions

## temporal locality

“if a value is accessed now, it will be accessed again soon”

caches should keep **recently accessed values**

## spatial locality

“if a value is accessed now, adjacent values will be accessed soon”

caches should **store adjacent values at the same time**

natural properties of programs — think about loops

# adding associativity

2-way set associative, 2 byte blocks, 2 sets

index	valid	tag	value	valid	tag	value
0	0			0		
1	0			0		

multiple places to put values with same index  
avoid conflict misses

# adding associativity

2-way set associative, 2 byte blocks, 2 sets

index	valid	tag	value	valid	tag	value
0	0		set 0	0		
1	0		set 1	0		

# adding associativity

2-way set associative, 2 byte blocks, 2 sets

index	valid	tag	value	valid	tag	value
0	0			0		
1	0			0		

way 0

way 1

# adding associativity

2-way set associative, 2 byte blocks, 2 sets

index	valid	tag	value	valid	tag	value
0	0			0		
1	0			0		

$m = 8$  bit addresses

$S = 2 = 2^s$  sets

$s = 1$  (set) index bits

$B = 2 = 2^b$  byte block size

$b = 1$  (block) offset bits

$t = m - (s + b) = 6$  tag bits

# adding associativity

2-way set associative, 2 byte blocks, 2 sets

index	valid	tag	value	valid	tag	value
0	1	000000	mem[0x00] mem[0x01]	0		
1	0			0		

address (hex)	result
00000000 (00)	miss
00000001 (01)	
01100011 (63)	
01100001 (61)	
01100010 (62)	
00000000 (00)	
01100100 (64)	

tag index offset

# adding associativity

2-way set associative, 2 byte blocks, 2 sets

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0	1	000000	mem[0x00] mem[0x01]	0		
1	0			0		

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00000000 (00)	miss
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01100100 (64)	

tag index offset



# adding associativity

2-way set associative, 2 byte blocks, 2 sets

index	valid	tag	value	valid	tag	value
0	1	000000	mem[0x00] mem[0x01]	0		
1	1	011000	mem[0x62] mem[0x63]	0		

address (hex)	result
00000000 (00)	miss
00000001 (01)	hit
01100011 (63)	miss
01100001 (61)	
01100010 (62)	
00000000 (00)	
01100100 (64)	

tag index offset

# adding associativity

2-way set associative, 2 byte blocks, 2 sets

index	valid	tag	value	valid	tag	value
0	1	000000	mem[0x00] mem[0x01]	1	011000	mem[0x60] mem[0x61]
1	1	011000	mem[0x62] mem[0x63]	0		

address (hex)	result
00000000 (00)	miss
00000001 (01)	hit
01100011 (63)	miss
01100001 (61)	miss
01100010 (62)	
00000000 (00)	
01100100 (64)	

tag index offset

# adding associativity

2-way set associative, 2 byte blocks, 2 sets

index	valid	tag	value	valid	tag	value
0	1	000000	mem[0x00] mem[0x01]	1	011000	mem[0x60] mem[0x61]
1	1	011000	mem[0x62] mem[0x63]	0		

address (hex)	result
00000000 (00)	miss
00000001 (01)	hit
01100011 (63)	miss
01100001 (61)	miss
01100010 (62)	hit
00000000 (00)	
01100100 (64)	

tag index offset

# adding associativity

2-way set associative, 2 byte blocks, 2 sets

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1	1	011000	mem[0x62] mem[0x63]	0		

address (hex)	result
00000000 (00)	miss
00000001 (01)	hit
01100011 (63)	miss
01100001 (61)	miss
01100010 (62)	hit
00000000 (00)	hit
01100100 (64)	

tag index offset

# adding associativity

2-way set associative, 2 byte blocks, 2 sets

index	valid	tag	value	valid	tag	value
0	1	000000	mem[0x00] mem[0x01]	1	011000	mem[0x60] mem[0x61]
1	1	011000	mem[0x62] mem[0x63]	0		

address (hex)	result
00000000 (00)	miss
00000001 (01)	hit
01100011 (63)	miss
01100001 (61)	miss
01100010 (62)	hit
00000000 (00)	hit
01100100 (64)	miss

needs to replace block in set 0!

tag index offset

# adding associativity

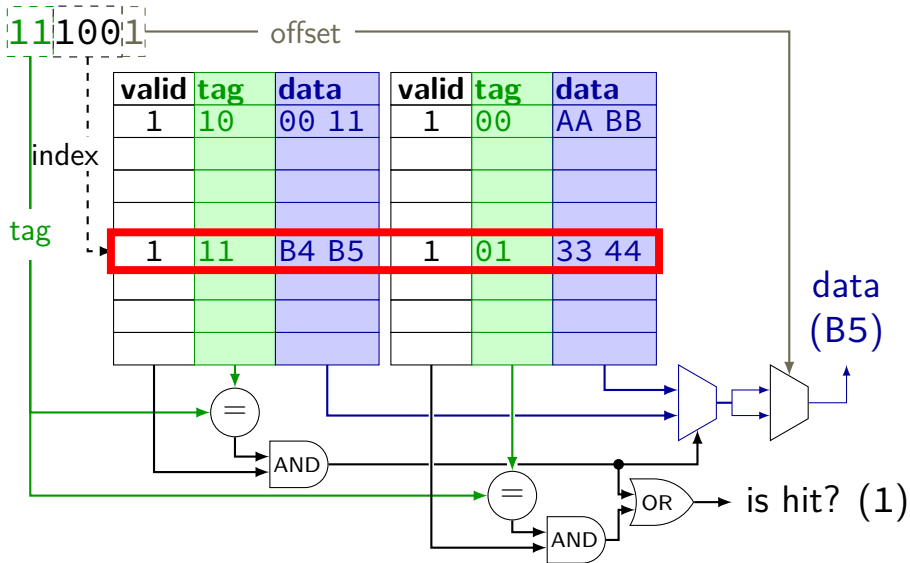
2-way set associative, 2 byte blocks, 2 sets

index	valid	tag	value	valid	tag	value
0	1	000000	mem[0x00] mem[0x01]	1	011000	mem[0x60] mem[0x61]
1	1	011000	mem[0x62] mem[0x63]	0		

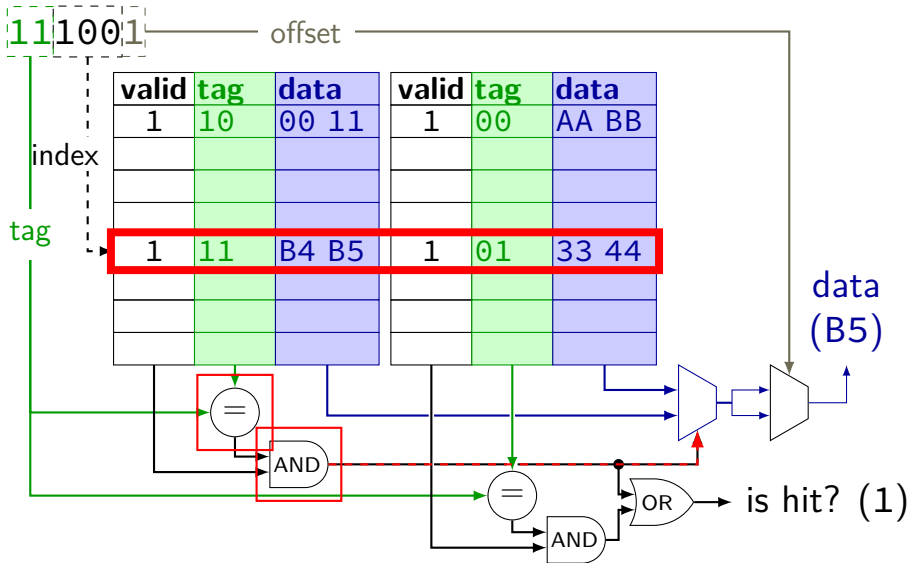
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00000000 (00)	miss
00000001 (01)	hit
01100011 (63)	miss
01100001 (61)	miss
01100010 (62)	hit
00000000 (00)	hit
01100100 (64)	miss

tag index offset

# cache operation (associative)

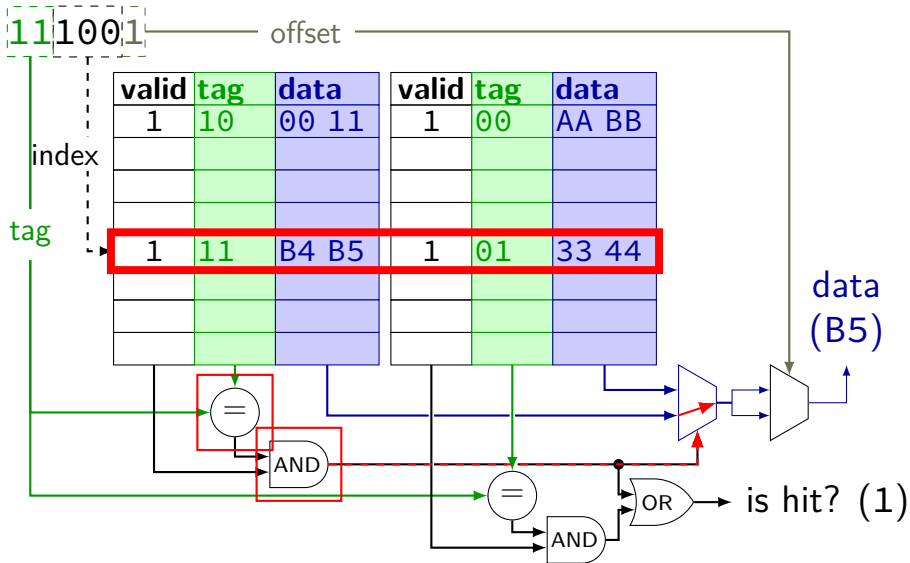


# cache operation (associative)





# cache operation (associative)



# associative lookup possibilities

none of the blocks for the index are valid

none of the valid blocks for the index match the tag  
something else is stored there

one of the blocks for the index is valid and matches the tag

# associativity terminology

**direct-mapped** — one block per set

***E*-way set associative** — *E* blocks per set  
*E* ways in the cache

**fully associative** — one set total (everything in one set)

# Tag-Index-Offset formulas (complete)

$m$  memory addresses bits (Y86-64: 64)

$E$  number of blocks per set (“ways”)

$S = 2^s$  number of sets

$s$  (set) index bits

$B = 2^b$  block size

$b$  (block) offset bits

$t = m - (s + b)$  tag bits

$C = B \times S \times E$  cache size (excluding metadata)

# Tag-Index-Offset exercise

$m$	memory addresses bits (Y86-64: 64)
$E$	number of blocks per set (“ways”)
$S = 2^s$	number of sets
$s$	(set) index bits
$B = 2^b$	block size
$b$	(block) offset bits
$t = m - (s + b)$	tag bits
$C = B \times S \times E$	cache size (excluding metadata)

## My desktop:

L1 Data Cache: 32 KB, 8 blocks/set, 64 byte blocks

L2 Cache: 256 KB, 4 blocks/set, 64 byte blocks

L3 Cache: 8 MB, 16 blocks/set, 64 byte blocks

Divide the address 0x34567 into **tag**, **index**, **offset** for each cache.

# T-I-O exercise: L1

quantity

value for L1

---

block size (given)

$B = 64\text{Byte}$

---

$B = 2^b$  ( $b$ : block offset bits)

# T-I-O exercise: L1

quantity	value for L1
block size (given)	$B = 64\text{Byte}$
	$B = 2^b$ ( $b$ : block offset bits)
block offset bits	$b = 6$

# T-I-O exercise: L1

quantity	value for L1
block size (given)	$B = 64\text{Byte}$
	$B = 2^b$ ( $b$ : block offset bits)
block offset bits	$b = 6$
blocks/set (given)	$E = 8$
cache size (given)	$C = 32\text{KB} = E \times B \times S$



# T-I-O exercise: L1

quantity	value for L1
block size (given)	$B = 64\text{Byte}$
	$B = 2^b$ ( $b$ : block offset bits)
block offset bits	$b = 6$
blocks/set (given)	$E = 8$
cache size (given)	$C = 32\text{KB} = E \times B \times S$
	$S = \frac{C}{B \times E}$ ( $S$ : number of sets)

# T-I-O exercise: L1

quantity	value for L1
block size (given)	$B = 64\text{Byte}$
	$B = 2^b$ ( $b$ : block offset bits)
block offset bits	$b = 6$
blocks/set (given)	$E = 8$
cache size (given)	$C = 32\text{KB} = E \times B \times S$
	$S = \frac{C}{B \times E}$ ( $S$ : number of sets)
number of sets	$S = \frac{32\text{KB}}{64\text{Byte} \times 8} = 64$

# T-I-O exercise: L1

quantity	value for L1
block size (given)	$B = 64\text{Byte}$
	$B = 2^b$ ( $b$ : block offset bits)
block offset bits	$b = 6$
blocks/set (given)	$E = 8$
cache size (given)	$C = 32\text{KB} = E \times B \times S$
	$S = \frac{C}{B \times E}$ ( $S$ : number of sets)
number of sets	$S = \frac{32\text{KB}}{64\text{Byte} \times 8} = 64$
	$S = 2^s$ ( $s$ : set index bits)
set index bits	$s = \log_2(64) = 6$

## T-I-O results

	L1	L2	L3
sets	64	1024	8192
block offset bits	6	6	6
set index bits	6	10	13
tag bits		(the rest)	

## T-I-O: splitting

	L1	L2	L3		
block offset bits	6	6	6		
set index bits	6	10	13		
tag bits	(the rest)				
0x34567:	3	4	5	6	7
	0011	0100	0101	0110	0111
bits 0-5 (all offsets):	100111 = 0x27				

## T-I-O: splitting

	L1	L2	L3		
block offset bits	6	6	6		
set index bits	6	10	13		
tag bits	(the rest)				
0x34567:	3	4	5	6	7
	0011	0100	0101	0110	0111
bits 0-5 (all offsets):	100111 = 0x27				

## T-I-O: splitting

	L1	L2	L3
block offset bits	6	6	6
set index bits	6	10	13
tag bits	(the rest)		

0x34567:	3	4	5	6	7
	0011	0100	0101	0110	0111

bits 0-5 (all offsets): 100111 = 0x27

L1:

bits 6-11 (L1 set): 01 0101 = 0x15

bits 12- (L1 tag): 0x34

# T-I-O: splitting

	L1	L2	L3
block offset bits	6	6	6
set index bits	6	10	13
tag bits	(the rest)		

0x34567:      3            4            5            6            7  
                 0011    0100    0101    0110    0111

bits 0-5 (all offsets): 100111 = 0x27

L1:

bits 6-11 (L1 set): 01 0101 = 0x15

bits 12- (L1 tag): 0x34



## T-I-O: splitting

	L1	L2	L3
block offset bits	6	6	6
set index bits	6	10	13
tag bits	(the rest)		

0x34567:      3            4            5            6            7  
                 0011    0100    0101    0110    0111

bits 0-5 (all offsets): 100111 = 0x27

L2:

bits 6-15 (set for L2): 01 0001 0101 = 0x115

bits 16-: 0x3

## T-I-O: splitting

	L1	L2	L3
block offset bits	6	6	6
set index bits	6	10	13
tag bits	(the rest)		

0x34567:	3	4	5	6	7
	0011	0100	0101	0110	0111

bits 0-5 (all offsets): 100111 = 0x27

L2:

bits 6-15 (set for L2): 01 0001 0101 = 0x115

bits 16-: 0x3

## T-I-O: splitting

	L1	L2	L3
block offset bits	6	6	6
set index bits	6	10	13
tag bits	(the rest)		

0x34567:      3            4            5            6            7  
          0011    0100    0101    0110    0111

bits 0-5 (all offsets): 100111 = 0x27

L3:

bits 6-18 (set for L3): 0 1101 0001 0101 = 0xD15

bits 18-: 0x0

# cache miss types

*compulsory* (or *cold*) — **first time** accessing something  
doesn't matter how big/flexible the cache is

*conflict* — sets aren't big/flexible enough  
a fully-associative (1-set) cache of the same size would have done better

*capacity* — cache was not big enough

# replacement policies

2-way set associative, 2 byte blocks, 2 sets

index	valid	tag	value	valid	tag	value
0	1	000000	mem[0x00] mem[0x01]	1	011000	mem[0x60] mem[0x61]
1	1	011000	mem[0x62] mem[0x63]	0		

address (hex)	result
000	
00000001 (01)	hit
01100011 (63)	miss
01100001 (61)	miss
01100010 (62)	hit
00000000 (00)	hit
01100100 (64)	miss

how to decide where to insert 0x64?

# replacement policies

2-way set associative, 2 byte blocks, 2 sets

index	valid	tag	value	valid	tag	value	LRU
0	1	000000	mem[0x00] mem[0x01]	1	011000	mem[0x60] mem[0x61]	1
1	1	011000	mem[0x62] mem[0x63]	0			1

address (hex)	result
00000000 (00)	miss
00000001 (01)	hit
01100011 (63)	miss
01100001 (61)	miss
01100010 (62)	hit
00000000 (00)	hit
01100100 (64)	miss

track which block was read least recently updated on **every access**

# example replacement policies

least recently used and approximations

take advantage of **temporal locality**

exact:  $\lceil \log_2(E!) \rceil$  bits per set for  $E$ -way cache

good approximations:  $E$  to  $2E$  bits

first-in, first-out

counter per set — where to replace next

(pseudo-)random

no extra information!

# exercise

4 byte blocks, 2 sets

index	V	tag	value	V	tag	value	LRU
0	0			0			
1	0			0			

address (hex)	hit?
00000000 (00)	
00000001 (01)	
00001010 (0A)	
00100001 (21)	
00001100 (0C)	
00000011 (02)	
00100011 (23)	



# exercise

4 byte blocks, 2 sets

index	V	tag	value	V	tag	value	LRU
0	0			0			
1	0			0			

address (hex)	hit?
00000000 (00)	
00000001 (01)	
00001010 (0A)	
00100001 (21)	
00001100 (0C)	
00000011 (02)	
00100011 (23)	

how is the address 21 (00100001) split up into tag/index/offset?

$b$  block offset bits;

$B = 2^b$  byte block size;

$s$  set index bits;  $S = 2^s$  sets;

$t = m - (s + b)$  tag bits (leftover)

# exercise

4 byte blocks, 2 sets

index	V	tag	value	V	tag	value	LRU
0	0			0			
1	0			0			

address (hex)	hit?
00000000 (00)	
00000001 (01)	
00001010 (0A)	
00100001 (21)	
00001100 (0C)	
00000011 (02)	
00100011 (23)	

tag index offset

# exercise

4 byte blocks, 2 sets

index	V	tag	value	V	tag	value	LRU
0	0			0			
1	0			0			

address (hex)	hit?
00000000 (00)	
00000001 (01)	
00001010 (0A)	
00100001 (21)	
00001100 (0C)	
00000011 (02)	
00100011 (23)	

tag index offset

exercise: how many accesses are hits?  
what is the final state of the cache?

# exercise

4 byte blocks, 2 sets

index	V	tag	value	V	tag	value	LRU
0	1	000000	M[0x00] M[0x01] M[0x02] M[0x03]	0			way 1
1	0			0			

address (hex)	hit?
00000000 (00)	miss
00000001 (01)	
00001010 (0A)	
00100001 (21)	
00001100 (0C)	
00000011 (02)	
00100011 (23)	

tag index offset

exercise: how many accesses are hits?  
what is the final state of the cache?

# exercise

4 byte blocks, 2 sets

index	V	tag	value	V	tag	value	LRU
0	1	000000	M[0x00] M[0x01] M[0x02] M[0x03]	1	000001	M[0x08] M[0x09] M[0x0A] M[0x0B]	way 0
1	0			0			

address (hex)	hit?
00000000 (00)	miss
00000001 (01)	hit
00001010 (0A)	miss
00100001 (21)	
00001100 (0C)	
00000011 (02)	
00100011 (23)	

tag index offset

# exercise

4 byte blocks, 2 sets

index	V	tag	value	V	tag	value	LRU
0	1	00100	M[0x20] M[0x21] M[0x22] M[0x23]	1	00001	M[0x08] M[0x09] M[0x0A] M[0x0B]	way 1
1	0			0			

address (hex)	hit?
00000000 (00)	miss
00000001 (01)	hit
00001010 (0A)	miss
00100001 (21)	miss
00001100 (0C)	miss
00000011 (02)	
00100011 (23)	

tag index offset

# exercise

4 byte blocks, 2 sets

index	V	tag	value	V	tag	value	LRU
0	1	00100	M[0x20] M[0x21] M[0x22] M[0x23]	1	00000	M[0x00] M[0x01] M[0x02] M[0x03]	way 0
1	1	00000	M[0x0C] M[0x0D] M[0x0E] M[0x0F]	0			way 1

address (hex)	hit?
00000000 (00)	miss
00000001 (01)	hit
00001010 (0A)	miss
00100001 (21)	miss
00001100 (0C)	miss
00000011 (02)	miss
00100011 (23)	

tag index offset

# exercise

4 byte blocks, 2 sets

index	V	tag	value	V	tag	value	LRU
0	1	00100	M[0x20] M[0x21] M[0x22] M[0x23]	1	00000	M[0x00] M[0x01] M[0x02] M[0x03]	way 1
1	1	00000	M[0x0C] M[0x0D] M[0x0E] M[0x0F]	0			way 1

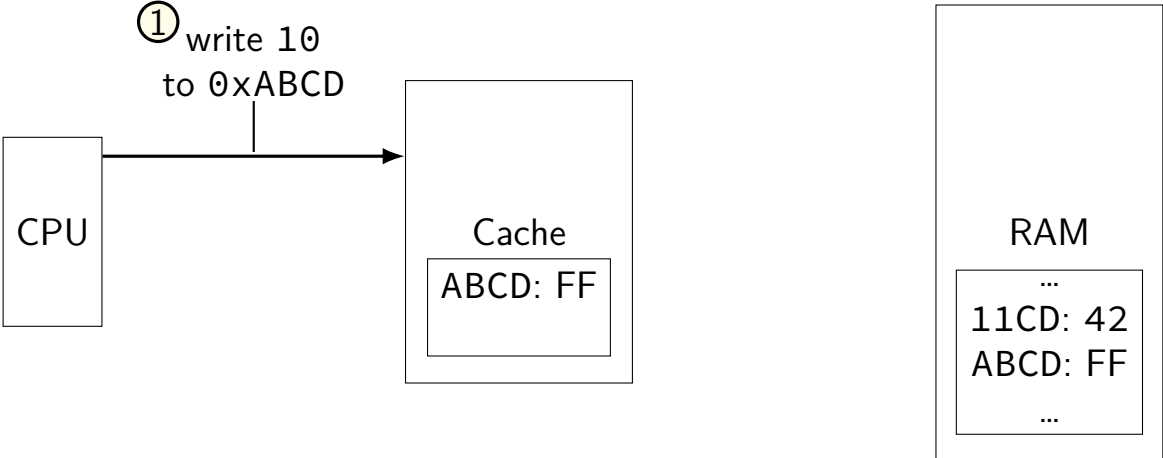
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00000001 (01)	hit
00001010 (0A)	miss
00100001 (21)	miss
00001100 (0C)	miss
00000011 (02)	miss
00100011 (23)	hit

tag index offset



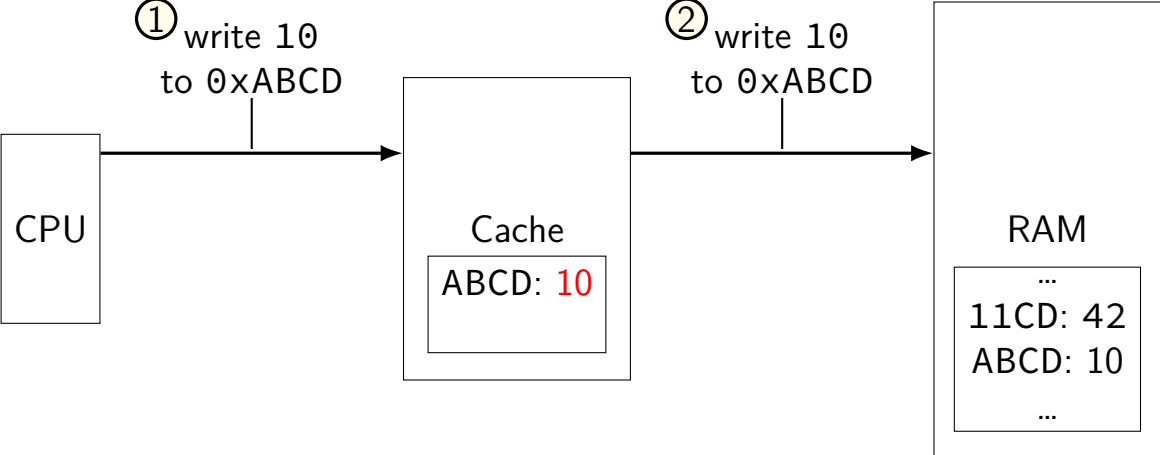
# write-through v. write-back

## option 1: write-through



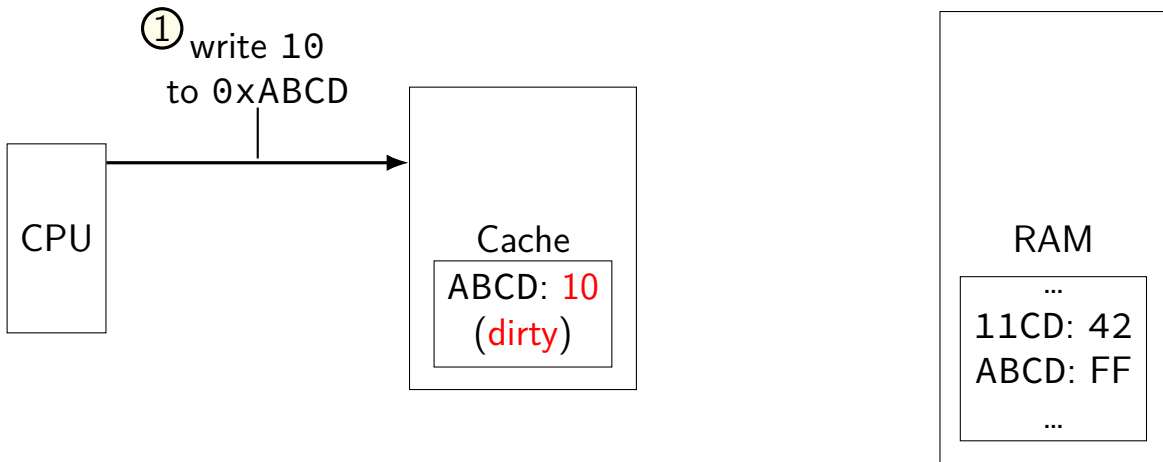
# write-through v. write-back

## option 1: write-through



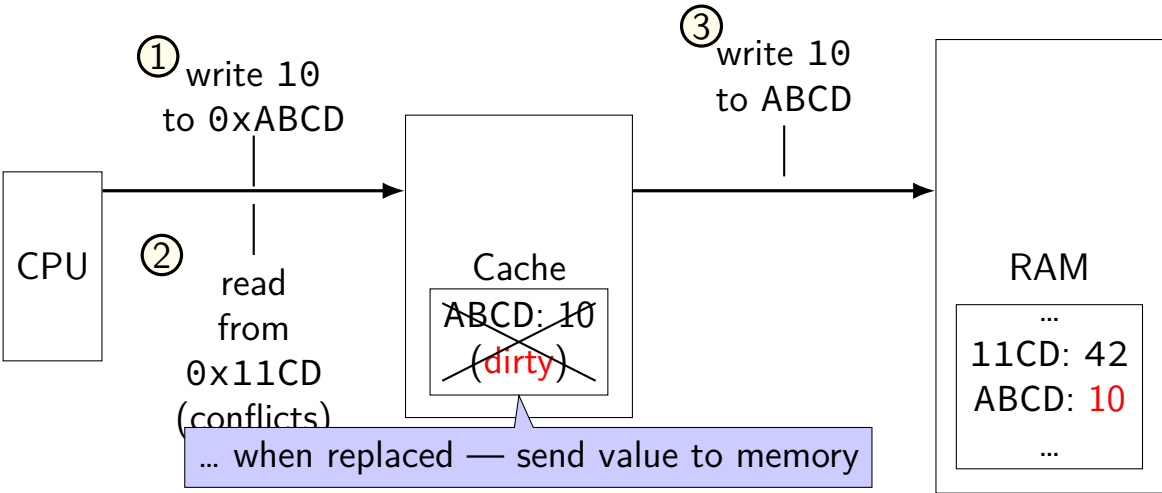
# write-through v. write-back

## option 2: write-back

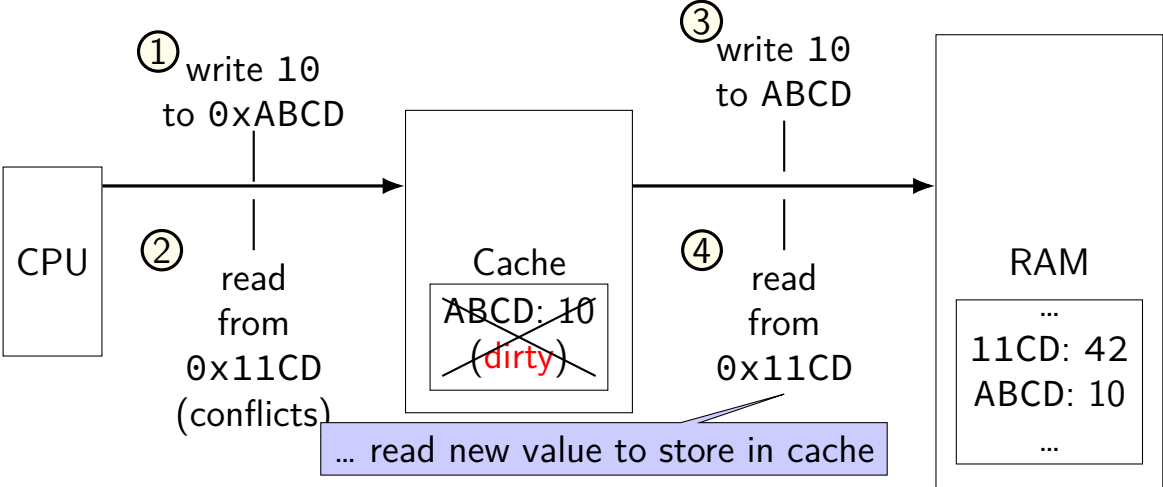


# write-through v. write-back

## option 2: write-back



# write-through v. write-back



# writeback policy

changed value!

2-way set associative, 4 byte blocks, 2 sets

index	valid	tag	value	dirty	valid	tag	value	dirty	LRU
0	1	000000	mem[0x00] mem[0x01]	0	1	011000	mem[0x60]* mem[0x61]*	1	1
1	1	011000	mem[0x62] mem[0x63]	0	0				0

1 = dirty (different than memory)  
needs to be written if evicted

# allocate on write?

processor writes **less than whole** cache block

block not yet in cache

two options:

## write-allocate

fetch rest of cache block, replace written part

## write-no-allocate

send write through to memory

guess: not read soon?

# write-allocate

2-way set associative, LRU, writeback

index	valid	tag	value	dirty	valid	tag	value	dirty	LRU
0	1	000000	mem[0x00] mem[0x01]	0	1	011000	mem[0x60]* mem[0x61]*	1	1
1	1	011000	mem[0x62] mem[0x63]	0	0				0

writing 0xFF into address 0x04?

index 0, tag 000001



# write-allocate

2-way set associative, LRU, writeback

index	valid	tag	value	dirty	valid	tag	value	dirty	LRU
0	1	000000	mem[0x00] mem[0x01]	0	1	011000	mem[0x60]* mem[0x61]*	1	1
1	1	011000	mem[0x62] mem[0x63]	0	0				0

writing 0xFF into address 0x04?

index 0, tag 000001

step 1: find **least recently used** block

# write-allocate

2-way set associative, LRU, writeback

index	valid	tag	value	dirty	valid	tag	value	dirty	LRU
0	1	000000	mem[0x00] mem[0x01]	0	1	011000	mem[0x60]* mem[0x61]*	1	1
1	1	011000	mem[0x62] mem[0x63]	0	0				0

writing 0xFF into address 0x04?

index 0, tag 000001

step 1: find **least recently used** block

step 2: possibly writeback old block

# write-allocate

2-way set associative, LRU, writeback

index	valid	tag	value	dirty	valid	tag	value	dirty	LRU
0	1	000000	mem[0x00] mem[0x01]	0	1	011000	0xFF mem[0x05]	1	0
1	1	011000	mem[0x62] mem[0x63]	0	0				0

writing 0xFF into address 0x04?

index 0, tag 000001

step 1: find **least recently used** block

step 2: possibly writeback old block

step 3a: read in new block – to get mem[0x05]

step 3b: update LRU information

# write-no-allocate

2-way set associative, LRU, writeback

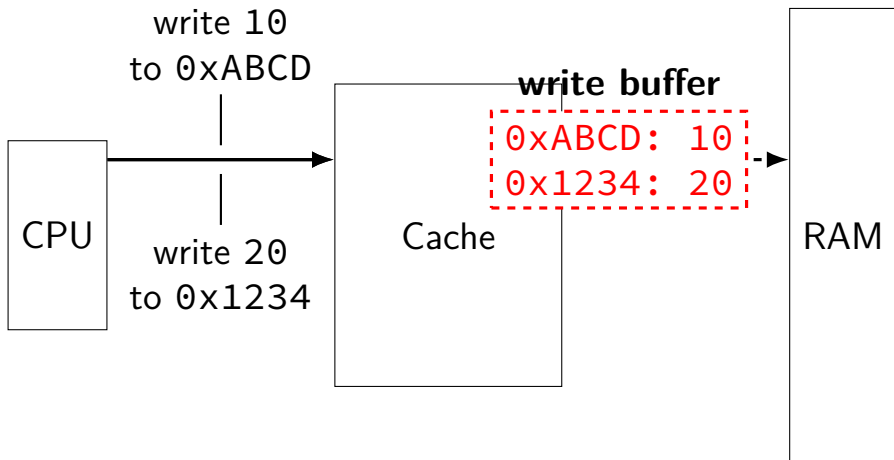
index	valid	tag	value	dirty	valid	tag	value	dirty	LRU
0	1	000000	mem[0x00] mem[0x01]	0	1	011000	mem[0x60]* mem[0x61]*	1	1
1	1	011000	mem[0x62] mem[0x63]	0	0				0

writing 0xFF into address 0x04?

step 1: is it in cache yet?

step 2: no, just send it to memory

# fast writes



write appears to complete immediately when placed in buffer  
memory can be much slower

# cache organization and miss rate

depends on program; one example:

SPEC CPU2000 benchmarks, 64B block size

LRU replacement policies

data cache miss rates:

Cache size	direct-mapped	2-way	8-way	fully assoc.
1KB	8.63%	6.97%	5.63%	5.34%
2KB	5.71%	4.23%	3.30%	3.05%
4KB	3.70%	2.60%	2.03%	1.90%
16KB	1.59%	0.86%	0.56%	0.50%
64KB	0.66%	0.37%	0.10%	0.001%
128KB	0.27%	0.001%	0.0006%	0.0006%

# cache organization and miss rate

depends on program; one example:

SPEC CPU2000 benchmarks, 64B block size

LRU replacement policies

data cache miss rates:

Cache size	direct-mapped	2-way	8-way	fully assoc.
1KB	8.63%	6.97%	5.63%	5.34%
2KB	5.71%	4.23%	3.30%	3.05%
4KB	3.70%	2.60%	2.03%	1.90%
16KB	1.59%	0.86%	0.56%	0.50%
64KB	0.66%	0.37%	0.10%	0.001%
128KB	0.27%	0.001%	0.0006%	0.0006%

# reasoning about cache performance

**hit time:** time to lookup and find value in cache

L1 cache — typically 1 cycle?

**miss rate:** portion of hits (value in cache)

**miss penalty:** extra time to get value if there's a miss  
time to access next level cache or memory

**miss time:** hit time + miss penalty



## average memory access time

$AMAT = \text{hit time} + \text{miss penalty} \times \text{miss rate}$

effective speed of memory

# making any cache look bad

1. access enough blocks, to fill the cache
2. access an additional block, replacing something
3. access last block replaced
4. access last block replaced
5. access last block replaced
- ...

but — typical real programs have **locality**

# cache optimizations

	miss rate	hit time	miss penalty
increase cache size	better	worse	—
increase associativity	better	worse	worse?
increase block size	depends	worse	worse
add secondary cache	—	—	better
write-allocate	better	—	worse?
writeback	???	—	worse?
LRU replacement	better	?	worse?

average time = hit time + miss rate × miss penalty

# cache optimizations by miss type

	capacity	conflict	compulsory
increase cache size	fewer misses	fewer misses	—
increase associativity	—	fewer misses	—
increase block size	—	more misses	fewer misses

(assuming other listed parameters remain constant)

## exercise (1)

initial cache: 64-byte blocks, 64 sets, 8 ways/set

If we leave the other parameters listed above unchanged, which will probably reduce the number of **capacity misses** in a typical program? (Multiple may be correct.)

- A. quadrupling the block size (256-byte blocks, 64 sets, 8 ways/set)
- B. quadrupling the number of sets
- C. quadrupling the number of ways/set

## exercise (2)

initial cache: 64-byte blocks, 8 ways/set, 64KB cache

If we leave the other parameters listed above unchanged, which will probably reduce the number of **capacity misses** in a typical program? (Multiple may be correct.)

- A. quadrupling the block size (256-byte block, 8 ways/set, 64KB cache)
- B. quadrupling the number of ways/set
- C. quadrupling the cache size

## exercise (3)

initial cache: 64-byte blocks, 8 ways/set, 64KB cache

If we leave the other parameters listed above unchanged, which will probably reduce the number of **conflict misses** in a typical program?

(Multiple may be correct.)

- A. quadrupling the block size (256-byte block, 8 ways/set, 64KB cache)
- B. quadrupling the number of ways/set
- C. quadrupling the cache size