Substitution

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Recurrence Solving Techniques

Tree

?✓Guess/Check

“Cookbook”

Substitution

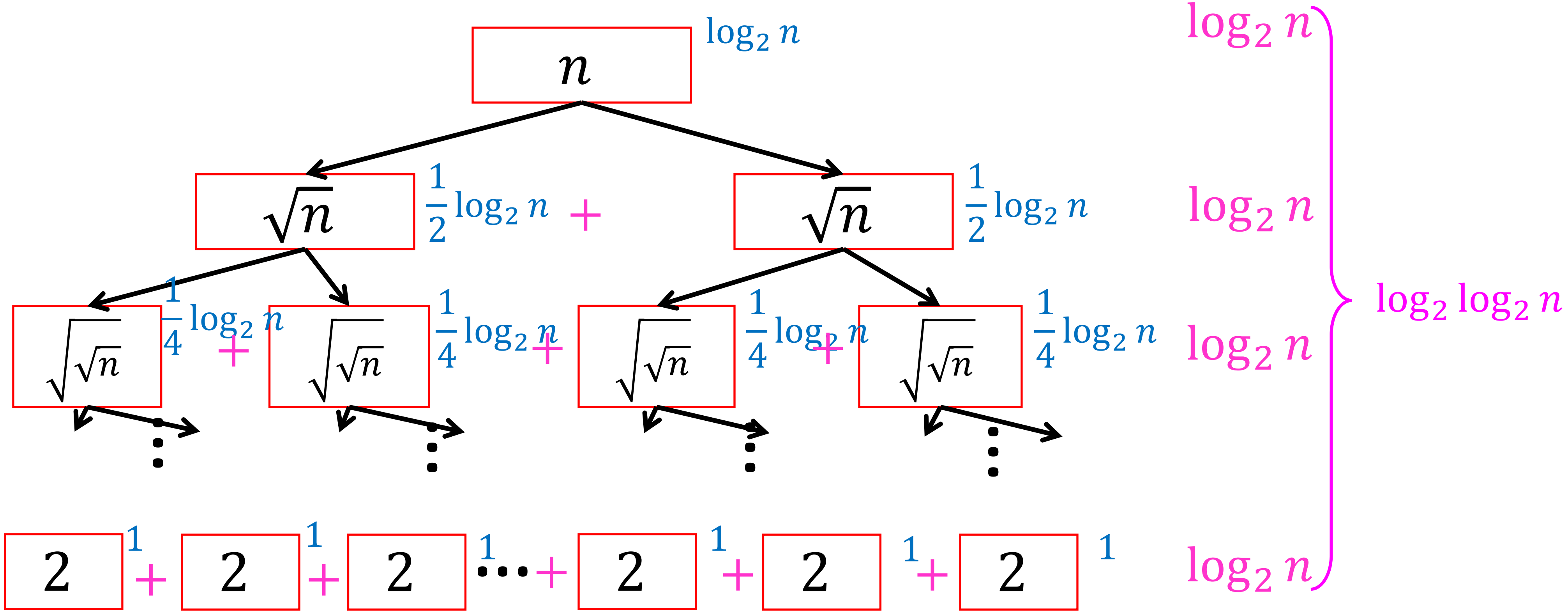
Substitution Method

- Idea: take a "difficult" recurrence, re-express it such that one of our other methods applies.
- Example:

$$T(n) = 2T(\sqrt{n}) + \log_2 n$$

Tree method

$$T(n) = 2T(\sqrt{n}) + \log_2 n$$



$$T(n) = O(\log_2 n \cdot \log_2 \log_2 n)$$

Substitution Method

- Idea: take a "difficult" recurrence, re-express it such that one of our other methods applies.
- Example: $T(n) = 2T(\sqrt{n}) + \log_2 n$

Let $n = 2^m$, i.e. $m = \log_2 n$

$T(2^m) = 2T\left(2^{\frac{m}{2}}\right) + m$ Rewrite in terms of exponent!

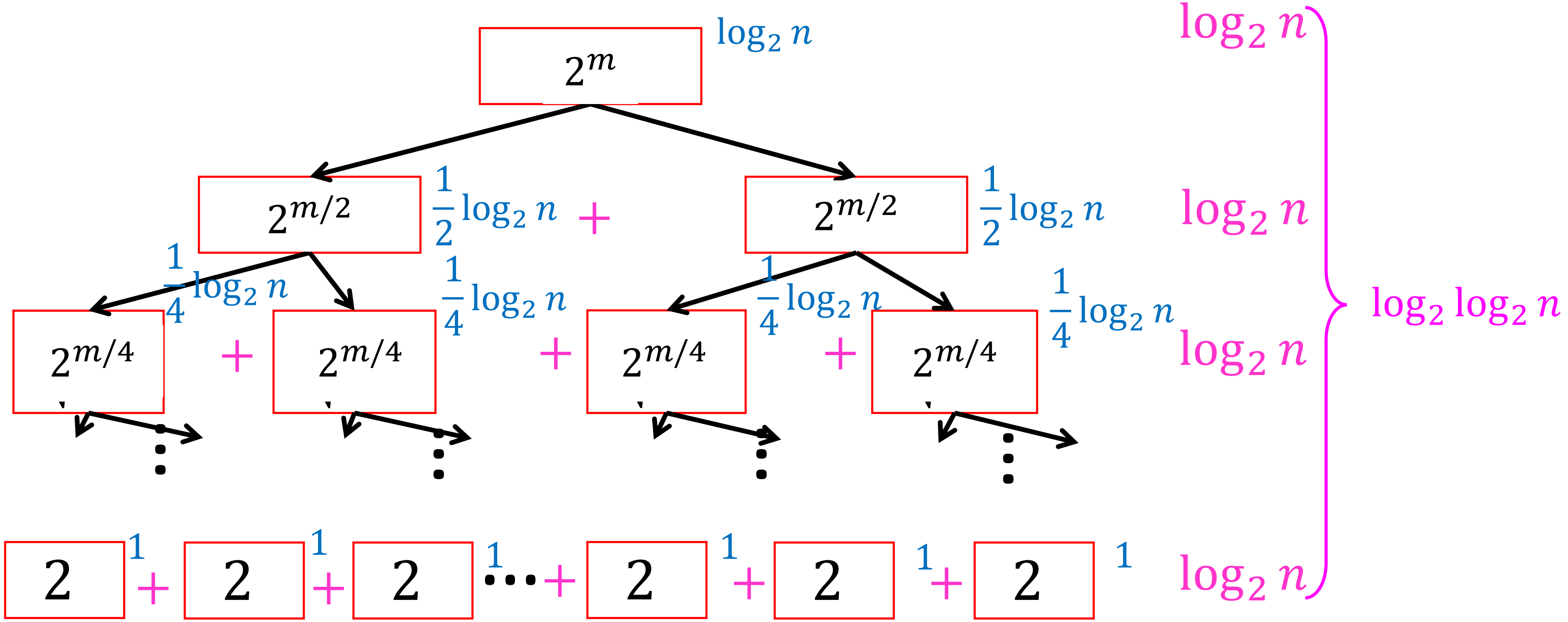
Let $S(m) = 2S\left(\frac{m}{2}\right) + m$ Case 2!

Let $S(m) = \Theta(m \log m)$ Substitute Back

Let $\mathrm{T}(n) = \Theta(\log n \log \log n)$

Tree method

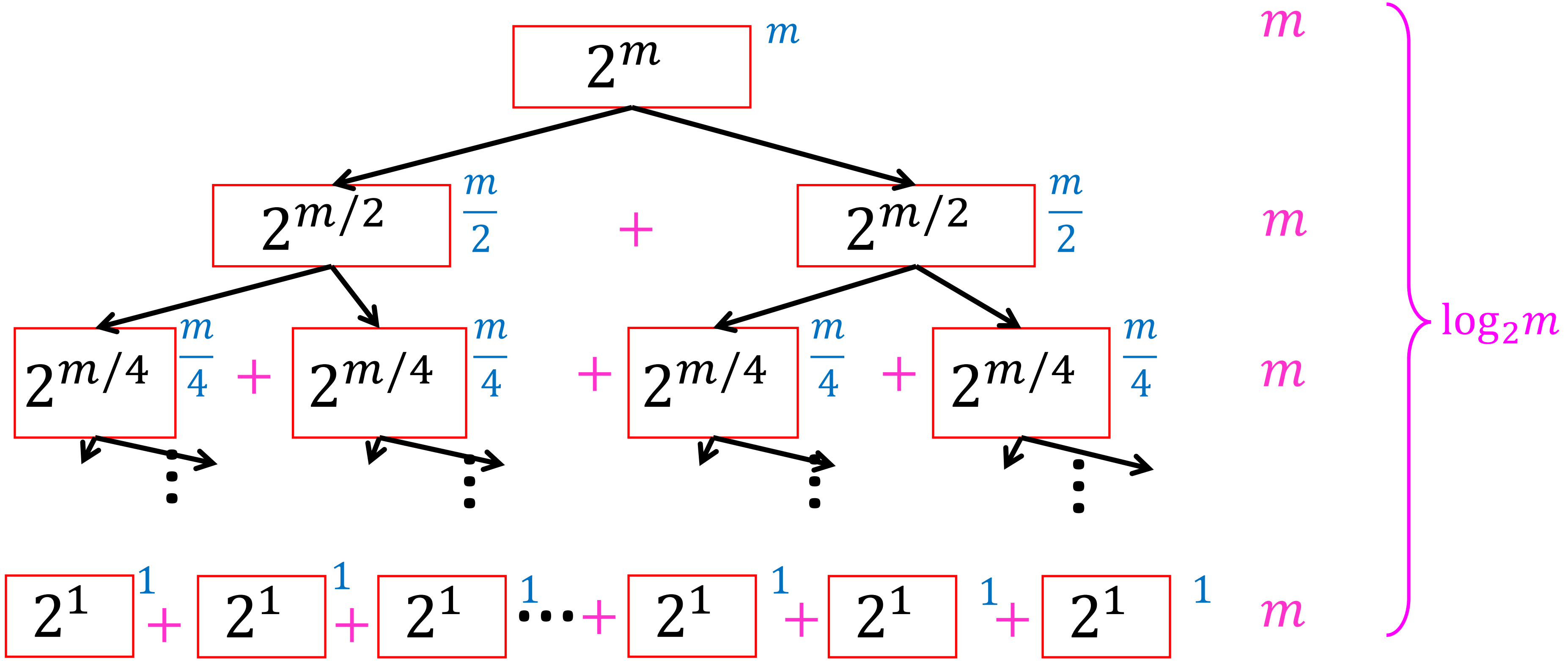
$n = 2^m$ $$T(2^m) = 2T\left(2^{\frac{m}{2}}\right) + m$$



Tree method

$n = 2^m$

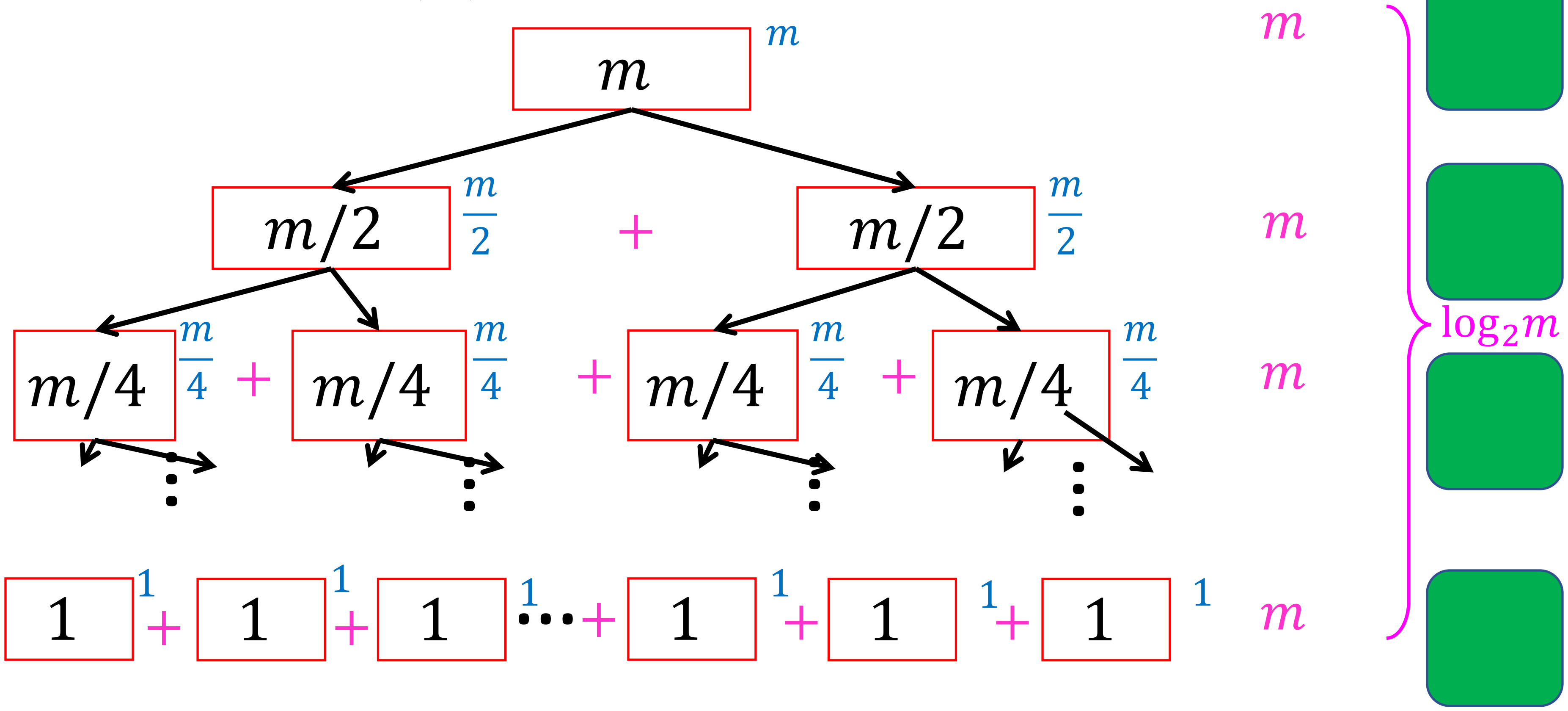
$T(2^m) = 2T(2^m) + \log_2 n$



Tree method

$$n = 2^m$$
$$T(2^m) = S(m)$$

$$S(m) = 2S\left(\frac{m}{2}\right) + m$$



$$T(n) = O(m \cdot \log_2 m) = O(\log_2 n \cdot \log_2 \log_2 n)$$